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The neutron total cross sections of some rare-earth elements between 0·7 MeV and 9·0 MeV

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Abstract. Measurements are presented of the total neutron cross sections of naturally occurring $_{57}$La, $_{58}$Ce, $_{59}$Pr, $_{60}$Nd, $_{62}$Sm, $_{64}$Gd, $_{66}$Dy, $_{67}$Ho, $_{68}$Er and $_{70}$Yb over the energy range 0·7 MeV to 9·0 MeV. The results are compared with theoretical cross sections evaluated using the spherical generalized optical potential of Wilmore and Hodgson. Considerable resonant structure is found in the total cross section of $_{58}$Ce and there is evidence of an abrupt onset of nuclear deformation as the neutron number reaches 90.

1. Introduction

The rare-earth nuclei extend over a region of the periodic table in which there is a marked variation in nuclear shape. The lightest members $_{57}$La, $_{58}$Ce and $_{59}$Pr of the group are almost spherical, being characterized by the closure of the major neutron shell with $N = 82$. Recent (p, t) reaction studies (Yagi *et al* 1972) of the even $_{60}$Nd isotopes indicate excited states which are vibrational in nature. Other two-neutron transfer reaction studies (Fleming *et al* 1973, Debenham and Hintz 1970) show that through the range of $_{62}$Sm and $_{64}$Gd isotopes the nuclear ground state shape changes from being almost spherical to being hard deformed and that vibrational, quasi-rotational and rotational excited states occur. The heavier members of the rare-earth nuclei mostly have ground states with large quadrupole moments and groupings of excited states showing strong rotational band structure.

In this paper we present measurements of the neutron total cross sections of naturally occurring $_{57}$La, $_{58}$Ce, $_{59}$Pr, $_{60}$Nd, $_{62}$Sm, $_{64}$Gd, $_{66}$Dy, $_{67}$Ho, $_{68}$Er and $_{70}$Yb between 0·7 MeV and 9·0 MeV obtained using the 100 m time of flight facility at the Kelvin Laboratory. The experimental cross sections are compared with the calculated values obtained using the spherical generalized optical potential of Wilmore and Hodgson (1964) in an attempt to establish whether any trends exist in the differences between experiment and theory as the nuclear deformation increases from $_{57}$La to $_{70}$Yb.

A similar study has been carried out by Glasgow and Foster (1970) over the energy range of 3·0 MeV to 15·0 MeV using natural targets. They found that the disagreement between the experimental data and the predicted values of the Perey and Buck non-local spherical, optical potential (Perey and Buck 1962) increases very sharply when the neutron number exceeds $N = 88$. Their measurements are consistent with calculations

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of Kumar and Baranger (1968) based on the pairing-plus-quadrupole model which predicts an abrupt onset of deformation as the neutron number reaches 90.

This paper makes a detailed comparison with the Glasgow and Foster data and other recent similar measurements, and investigates if the present experimental values for the total neutron cross sections from 0·7 MeV to 9·0 MeV show evidence of the sudden increase in nuclear deformation predicted by Kumar and Baranger.

2. Experimental procedure

Details of the experimental layout, detectors, energy calibration, monitoring and counting and analysing techniques can be found in a previous paper (Kellie *et al* 1972) on neutron total cross section measurements, and a general description of the time of flight facility at the Kelvin Laboratory (Crawford *et al* 1972).

The samples were all 100% chemically pure, approximately 60% opaque to neutrons, and were kept in thin, tight-fitting polythene bags filled with argon to prevent oxidation from the air. Exact duplicates of the polythene bags were placed in the target position during 'target out' runs to compensate for their presence during the 'target in' runs. To minimize the effects of air absorption, all but 6·0 m of the flight path was evacuated.

The accelerator was operated at 1000 pps. Its current was regulated to give count rates of about 300 counts/s with both the target in and target out. This meant that since the random background was the same in both 'target in' and 'target out' spectra, the effect of the background could be easily eliminated from the total cross section evaluation. To avoid pile-up corrections a multi-stop time conversion system, which has been previously described (Crawford *et al* 1972), was used.

The data points are presented at 2 keV intervals at 0·7 MeV and 75 keV intervals at 9·0 MeV with statistical errors of typically 2%. The other main sources of error came from the normalization factors between 'target in' and 'target out' runs, and the measurements of the thickness and density of the samples, the combined effect of which is estimated to be $\pm 3\%$.

The samples were right cylinders typically 3 cm in diameter and 5 cm in length†.

3. Experimental results

The experimental cross sections, together with the calculations based on the generalized optical potential of Wilmore and Hodgson are presented in figure 1. The total energy range is split into two sections, which extend from 0·7 MeV to 2·1 MeV and from 2·0 MeV to 9·0 MeV with approximately the same number of experimental points in each. The error bars shown, denote the statistical errors only. In figures 2 and 3 the results of previous measurements are compared with an average of the present data.

In general there is good agreement between the present data and that of Glasgow and Foster in both the shape and absolute magnitude of the cross sections. There are however localized differences of as much as 6% which are most noticeable for $_{57}$La and $_{58}$Ce. For $_{59}$Pr, $_{60}$Nd and $_{66}$Dy small normalization differences of 2%, 3% and 3·5%, which are all compatible with the combined accuracy of the results, are evident. For the remaining elements, $_{62}$Sm, $_{64}$Gd, $_{67}$Ho, $_{68}$Er and $_{70}$Yb the agreement with the

† We are indebted to AWRE Aldermaston for the loan of all the samples except La and Ce.

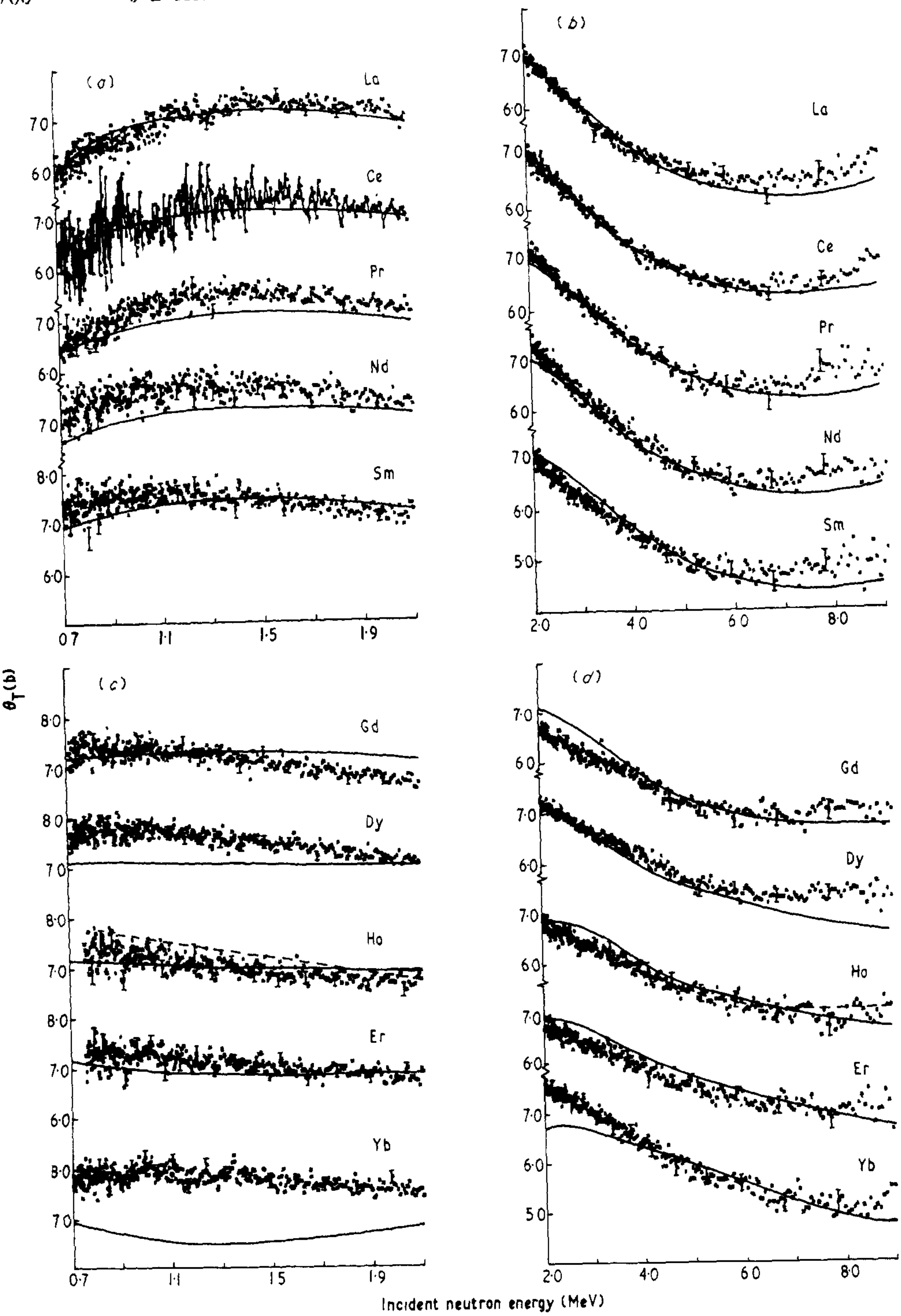

Figure 1. Total neutron cross-sections of (*a*) and (*b*) La, Ce, Pr, Nd and Sm; and (*c*) and (*d*) Gd, Dy, Ho, Er and Yb between 0·7 MeV and 9·0 MeV. The full line denotes the theoretical cross section evaluated using the spherical generalized potential of Wilmore and Hodgson and the broken line the coupled-channel calculation of Marshak for Ho.

Glasgow and Foster measurements is excellent. This is of particular significance for $_{70}$Yb, since Glasgow and Foster expressed doubts about the absolute value of the measurement due to its anomalously large deviation (nine standard deviations) from the predictions of the spherical, non-local, optical potential of Perey and Buck. As can be seen from figures 2 and 3 the present results from 0·7 to 2·1 MeV agree very well with other previous measurements for $_{57}$La, $_{58}$Ce, $_{62}$Sm, $_{64}$Gd, $_{66}$Dy and $_{67}$Ho. There is no previous measurement reported for the total neutron cross section of $_{68}$Er over this energy range. For $_{59}$Pr, the present results are (5–7)% lower than the values of Miller *et al* (1952) and Islam *et al* (1973) although at higher energies (see figure 2) they are in good agreement with those of Manero (1968) and Carlson and Barschall (1967). For $_{60}$Nd the present results are 3% above those of Okazaki *et al* (1954). Since they are also 3% higher than the cross section obtained by Glasgow and Foster, perhaps a 3% normalization error exists in the present measurement of the total neutron cross section of $_{60}$Nd. For $_{70}$Yb the results of Okazaki *et al* are 10% below both the present values and those of Glasgow and Foster and consequently lie closer to the spherical optical model predictions. Clearly, the existing measurements of the total neutron cross sections of rare-earth nuclei agree in most cases to within ±4% over the range of energies

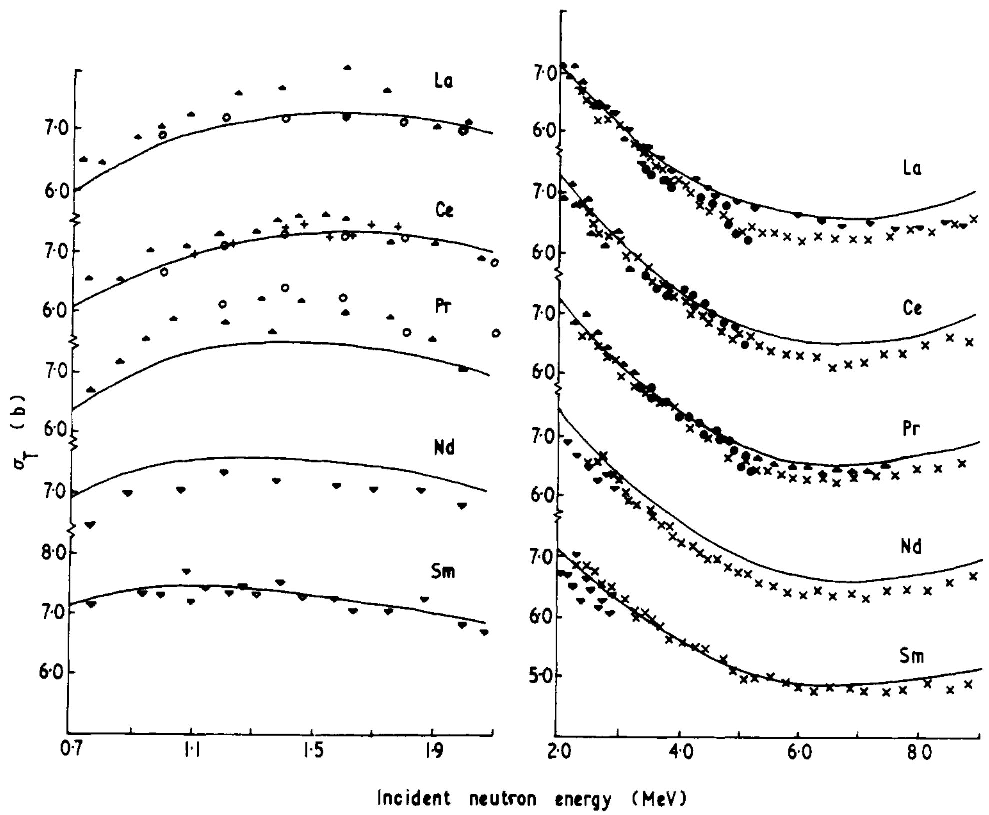

Figure 2. Comparison of an average of the present data (full line) with the results of other workers for the total neutron cross sections of La, Ce, Pr, Nd and Sm. In the figure, Δ refers to Miller *et al* (1952), ○ to Islam *et al* (1973), + to Wells *et al* (1963), ▽ to Okazaki *et al* (1954), × to Glasgow and Foster (1970), ▼ to Nereson and Darden (1954), ● to Manero (1968), and ▲ to Carlson and Barschall (1967).

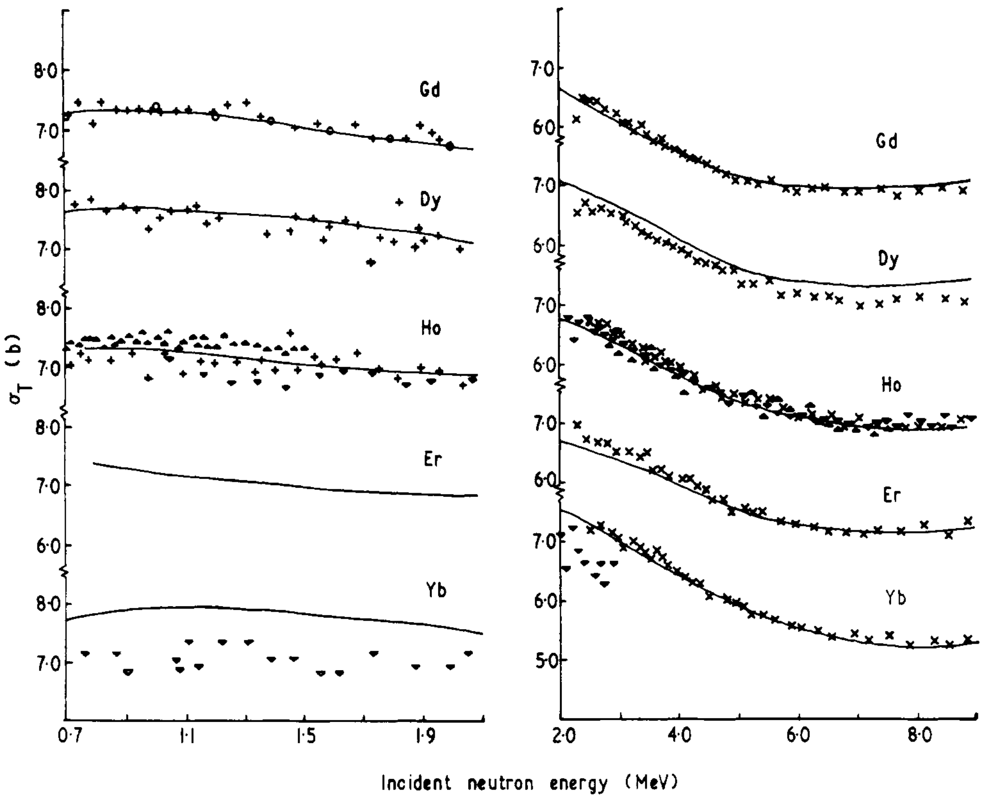

Figure 3. Comparison of an average of the present data (full line) with the results of other workers for the total neutron cross sections of Gd, Dy, Ho, Er and Yb. In the figure + refers to Stupegia (1966), ○ to Islam *et al* (1973), △ to Meadows *et al* (1971), ▼ to Marshak *et al* (1970), ▽ to Okazaki *et al* (1954), × to Glasgow and Foster (1970), and ▲ to Fasoli *et al* (1969).

being studied. However, there is still a requirement for further careful measurements to be made, particularly for $_{70}$Yb.

An examination of figure 1 shows qualitatively the agreement between the present experimental results and the calculated values obtained from the Wilmore and Hodgson potential. To quantify the analysis the following functions were evaluated for all the cross sections, ie

$$f = \left[\frac{1}{N}\sum_{E=E_i}^{E_f}\left(\frac{\sigma_e(E)-\sigma_t(E)}{\Delta\sigma_e(E)}\right)^2\right]^{1/2}$$

$$f_{\bar{n}} = \left[\frac{1}{N}\sum_{E=E_i}^{E_f}\left(\frac{\sigma_e(E)\bar{n}-\sigma_t(E)}{\Delta\sigma_e(E)}\right)^2\right]^{1/2}.$$

The sums extend over a specified energy range $(E_f - E_i)$ and have terms equispaced in energy.

In the formulae, N is the number of terms in summation, $\sigma_e(E)$ is the experimental cross section at energy E, $\sigma_t(E)$ is the theoretical cross section at energy E and $\Delta\sigma_e(E)$ is the statistical error in the experimental cross section at energy E.

$$\bar{n} = \left[\sum_{E=E_i}^{E_f}\left(\frac{\sigma_t(E)}{\Delta\sigma_e(E)}\right)^2\right]\left(\sum_{E=E_i}^{E_f}\frac{\sigma_e(E)\sigma_t(E)}{\Delta\sigma_e(E)^2}\right)^{-1}$$

is a quantity which tests for the presence of systematic and normalization errors. f examines the agreement between the magnitudes of the experimental and theoretical cross sections. A value of $f < 1$ implies that the average difference between the cross sections is less than the statistical error in the experimental data. $f_{\bar{n}}$ has the same form as f, but is modified to take account of any systematic errors in the experiment. An evaluation of $f_{\bar{n}}$ is only of significance if $|1-\bar{n}|$ is less than the total experimental error. For larger values the change in the magnitude of the experimental cross section cannot be justified and $f_{\bar{n}}$ assumes an unrealistically small value.

Three energy ranges were chosen, the complete range of the experiment from 0·7 MeV to 9·0 MeV with points taken at 50 keV intervals, 0·7 MeV to 3·0 MeV with points at 20 keV intervals and 3·0 MeV to 9·0 MeV with points at 50 keV intervals. 3·0 MeV was chosen as the dividing energy for the last two ranges to enable a direct comparison to be made with the results of Glasgow and Foster.

Values for f, $\bar{n}$ and $f_{\bar{n}}$ are listed in table 1. Also shown are the results obtained for $_{67}$Ho using a coupled-channel calculation performed by Marshak *et al* (1968). A coupled-channel calculation becomes necessary when a deformed optical potential is used to describe an interaction. In their calculation Marshak *et al* took a value of $\beta = 0{\cdot}33$ for the deformation parameter describing $_{67}$Ho and used the adiabatic approximation to reduce the computation time. The results for $\bar{n}$ indicate that, apart from Yb where values of $|1-\bar{n}|$ between 11% and 14% occur (so that $f_{\bar{n}}$ can be disregarded), normalizing factors greater than 4% are not required to improve the fits between theory and experiment. Since this is comparable with the total estimated error, the most probable values for the function f which can be deduced from the present data should lie between the listed values for f and $f_{\bar{n}}$. Another exception can probably be made for $_{67}$Ho where the extremely good agreement of several sets of experimental data over the range of 2·0 MeV to 9·0 MeV indicates that the systematic error in our measurement is about

Table 1. Values for the functions $\bar{n}$, $f_{\bar{n}}$ and f.

	(0·7–9·0) MeV 50 keV steps			(0·7–3·0) MeV 20 keV steps			(3·0–9·0) MeV 50 keV steps		
	$\bar{n}$	$f_{\bar{n}}$	f	$\bar{n}$	$f_{\bar{n}}$	f	$\bar{n}$	$f_{\bar{n}}$	f
La	1·003	3·25	3·26	1·00	3·10	3·10	1·017	2·31	2·44
Ce	0·990	3·33	3·64	0·985	3·70	4·36	1·010	1·69	1·78
Pr	0·980	2·15	2·88	0·975	1·85	3·18	1·004	1·46	1·46
Nd	0·973	2·82	3·96	0·966	2·61	4·53	1·004	1·53	1·53
Sm	1·008	2·98	3·03	1·006	2·85	2·90	1·015	2·15	2·23
Gd	1·039	4·08	5·61	1·040	4·33	6·08	1·035	2·33	2·95
Dy	0·961	4·19	6·79	0·957	4·44	7·74	0·979	2·08	2·63
Ho†	1·008	3·05	3·11	1·001	3·18	3·18	1·034	1·24	2·04
Ho‡	1·023	1·71	2·65	1·024	2·06	3·12	1·021	0·97	1·12
Er	0·998	3·35	3·36	0·990	3·47	3·66	1·024	1·30	1·77
Yb	0·890	5·93	13·51	0·867	4·05	16·07	0·978	1·41	1·87

† Refers to Wilmore and Hodgson potential.

‡ Refers to coupled-channel calculation of Marshak.

1 %. This implies that where $|1-\bar{n}|$ is greater than 1 % for any of the Ho cross sections an evaluation of $f_{\bar{n}}$ is not relevant and the comparison of theory and experiment is properly described by f.

4. Discussion

An examination of table 2 which lists the isotopic abundances of the naturally occurring rare-earth elements used in the present experiment shows that $^{139}_{57}$La, 88 % of $_{58}$Ce and $^{141}_{59}$Pr all have the common feature of a filled major neutron shell with $N = 82$. They differ in the number of protons outside the $Z = 50$ shell. Wildenthal (1969) has been able to predict correctly the spins and parities of the low-lying states of the above nuclei by assuming that the additional protons take up configurations of either m protons occupying the $1g_{7/2}$ and $2d_{5/2}$ orbitals or $(m-1)$ protons in these orbitals and 1 proton in either a $3s_{1/2}$ or $2d_{3/2}$ state. In spite of the similarity in the structure of the three nuclei it is seen from table 1 that between 0·7 MeV and 3·0 MeV, f and $f_{\bar{n}}$ are larger for $_{58}$Ce. This can be attributed to the existence of structure (see figure 1) in the measured total cross section of $_{58}$Ce up to about 2 MeV. The occurrence of the structure was investigated further by re-measuring the total cross section of $_{58}$Ce in the extended neutron energy range down to 250 keV. At these lower energies, many well separated resonances were evident. Although a more detailed analysis of these resonances will be included in a later paper it is interesting to point out that the reduction in the level density of $^{141}_{58}$Ce relative to $^{140}_{57}$La and $^{142}_{59}$Pr (the neutron total cross sections of $^{139}_{57}$La and $^{141}_{59}$Pr show little evidence of structure) provides a good example of the effect of nucleon pairing on level densities. The odd proton in $^{140}_{57}$La and $^{142}_{59}$Pr supplies the extra degree of freedom necessary to increase the level density. The level density of the $^{142}_{58}$Ce which constitutes 12 % of the natural cerium target may also be expected to be low by similar arguments.

For the 3·0 MeV to 9·0 MeV range there is a sharp increase in both f and $f_{\bar{n}}$ between $_{60}$Nd and $_{62}$Sm. This is in agreement with the results of Glasgow and Foster and probably can be interpreted as revealing the sudden onset of nuclear deformation when the neutron number exceeds 88. Further evidence of this abrupt transition region is found from (p, t) reactions at 18 MeV on the even $_{64}$Gd isotopes (Fleming *et al* 1973). For $^{160,158,156}_{64}$Gd(p, t) a strong ground state transition is observed. However in

Table 2. Isotopic composition of naturally occurring nuclei.

$_{57}$La	N	82						
	%	100						
$_{58}$Ce	N	82	84					
	%	88	12					
$_{59}$Pr	N	82						
	%	100						
$_{60}$Nd	N	82	83	84	85	86	88	90
	%	27	12	24	8	17	6	5
$_{62}$Sm	N	85	86	87	88	90	92	
	%	15	11	14	7	27	23	

$_{64}$Gd	N	91	92	93	94	95
	%	15	20	16	25	22
$_{66}$Dy	N	95	96	97	98	
	%	19	26	25	28	
$_{67}$Ho	N	98				
	%	100				
$_{68}$Er	N	98	99	100	102	
	%	33	23	27	15	
$_{70}$Yb	N	101	102	103	104	106
	%	14	22	16	32	13

$^{154}_{64}Gd_{90}(p, t)^{152}_{64}Gd_{88}$, which crosses the transition region, the ground state strength is reduced by a factor of two. This is thought to arise from the change from a deformed to an almost spherical ground state nucleus resulting in a poor overlap between the initial and final wavefunctions describing the interaction.

The variation of f and $f_{\bar{n}}$ between 0·7 MeV and 3·0 MeV over the transition region is not so abrupt. This may be a consequence of only 50% of naturally occurring $_{62}$Sm having N greater than 88. Also, from table 1 it can be seen that the values for f and $f_{\bar{n}}$ are larger than those for the 3·0 MeV to 9·0 MeV range due principally to the failure of the theory to reproduce not only the magnitude but also the shape of the cross sections, ie for $_{57}$La the data points lie below the theoretical curve at 0·7 MeV and above the curve at 2·1 MeV, whereas for $_{62}$Sm and $_{64}$Gd the opposite feature is observed.

The results for $_{67}$Ho are of particular interest since they illustrate how the fit to the data can be improved by using a coupled-channel calculation. This is particularly marked between 3·0 MeV and 9·0 MeV where f is reduced from 2·04 to 1·12. Between 0·7 MeV and 3·0 MeV the result is not so convincing but, as an examination of figure 1(*d*) shows, the shape of the cross section obtained from the coupled-channel calculation does bear a closer resemblance to the data.

In order to examine more thoroughly the correspondence in shape between the experimental and theoretical curves the following function was evaluated for all the elements over the range 0·7 to 9·0 MeV using 50 keV steps:

$$g = \left[\frac{1}{N}\sum_{E=E_i}^{E_f}\left(\frac{2(g_e^s(E)-g_t(E))}{|g_e^s(E)|+|g_t(E)|}\right)^2\right]^{1/2}$$

where $g_e^s(E)$ is the gradient of the smoothed experimental cross section at energy E and $g_t(E)$ is the gradient of the theoretical cross section at energy E.

It was hoped that the above analysis would be able to determine the significance of the large values for f and $f_{\bar{n}}$ obtained for $_{57}$La compared to neighbouring nuclei and of the relatively low values obtained for $_{67}$Ho and $_{66}$Er.

The values obtained for g are listed in table 3. It can be seen that the results fall into two groups: (0·96 to 1·04) for $_{57}$La, $_{58}$Ce and $_{59}$Pr and (1·16 to 1·30) for $_{62}$Sm to $_{70}$Yb with $_{60}$Nd having an intermediate value of 1·09. It is interesting to note that the two groups correspond with the expected division of the rare-earth nuclei into almost spherical nuclei and hard deformed nuclei. The coupled-channel calculation for $_{67}$Ho obtained a lower value of g than the spherical potential calculation, corresponding to

Table 3. Values for the function *g*.

	g		g
La	1·00 spherical	Gd	1·23 hard deformed
Ce	0·96 spherical	Dy	1·16 hard deformed
Pr	1·04 spherical	Ho†	1·16 hard deformed
		Er	1·19 hard deformed
Nd	1·09 vibrational	Yb	1·30 hard deformed
Sm	1·21 hard deformed	Ho‡	1·14 hard deformed

† Refers to Wilmore and Hodgson potential.
‡ Refers to coupled-channel calculation of Marshak.

an improved agreement between the shapes of the theoretical and experimental cross sections.

5. Conclusion

The present survey of the neutron total cross sections of a selection of the rare-earth nuclei has revealed several interesting features, in particular the occurrence of structure in the cross section of $_{58}$Ce up to 2 MeV and the onset of nuclear deformation when N exceeds 88.

However it is evident that for most of the nuclei studied, a spherical optical potential is unable to predict the correct shape for the cross sections. As was seen for $_{67}$Ho the magnitude and shape of the cross section curve are improved by using a coupled-channel calculation. Obviously, a coupled-channel analysis should be applied to the complete range of rare-earth nuclei.

It is hoped to attempt such an analysis in this laboratory in the future.

References

Carlson A D and Barschall H H 1967 *Phys. Rev.* **158** 1142–56
Crawford *et al* 1972 *Nucl. Instrum. Meth.* **109** 479–91
Debenham P and Hintz N 1970 *Phys. Rev. Lett.* **24** 44–6
Fasoli *et al* 1969 *Nucl. Phys.* A **133** 572–80
Fleming *et al* 1973 *Phys. Rev.* C **8** 806–18
Glasgow D W and Foster D G 1970 *Phys. Rev.* C **3** 604–20
Islam *et al* 1973 *Nucl. Phys.* A **209** 189–202
Kellie *et al* 1972 *J. Phys.* A: *Gen. Phys.* **5** 1090–4
Kumar K and Baranger M 1968 *Nucl. Phys.* A **110** 529–54
Manero R 1968 *Anales Real Soc. Espan. Fis. Quim.* **64** 63–70
Marshak *et al* 1968 *Phys. Rev. Lett.* **20** 554–8
Marshak *et al* 1970 *Phys. Rev.* C **2** 1862–79
Meadows *et al* 1971 *T.D.Z. Phys.* **243** 171–87
Miller *et al* 1952 *Phys. Rev.* **88** 83–90
Nereson N and Darden S 1954 *Phys. Rev.* **94** 1178–82
Okazaki *et al* 1954 *Phys. Rev.* **93** 461–3
Perey F and Buck B 1962 *Nucl. Phys.* **32** 353–80
Stupegia D C 1966 *Nuclear Constants* (Argonne National Laboratory)
Wells J T, Tucker A B and Meyerhof W E 1963 *Phys. Rev.* **131** 1644–60
Wildenthal B H 1969 *Phys. Rev. Lett.* **22** 1118–20
Wilmore D and Hodgson P E 1964 *Nucl. Phys.* **55** 673–94
Yagi *et al* 1972 *Phys. Rev. Lett.* **29** 1334–7